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MEAN-EXPECTED SHORTFALL PORTFOLIO OPTIMIZATION USING A GENETIC ALGORITHM

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Capital requirements for the market risk exposure of banks is a nonlinear function of the expected shortfall (ES), which is calculated based on a bank's actual portfolio, i.e. the portfolio represented by the bank's current holdings. To tackle portfolio optimization with respect to the ES, a genetic algorithm (GA) is used in this paper. The paper examines the effectiveness of a specific GA technique, namely the Strength Pareto Evolutionary Algorithm 2 (SPEA2) for portfolio optimization when the expected return (the mean) and percentage ES are set as the optimization goals. In addition, the differences between the mean-ES optimal portfolios and the mean-VaR optimal portfolios obtained by using the same optimization algorithm is analyzed in the study. The results document that the SPEA2 method provides well-distributed portfolios along the efficient frontier covering different risk levels. Compared to the mean-VaR optimal portfolios, the mean-ES optimal portfolios document superiority over the entire efficient frontier in the mean-ES plane. Concurrently, the converted mean-ES portfolios seem to converge towards the mean-VaR portfolios in the mean-VaR plane and nearly coincide for the high levels of the expected return.

Keywords: portfolio optimization, expected shortfall, VaR, SPEA2

JEL Classification: C61, C63, G11, G17, G21

INTRODUCTION

To ensure solvency for their clients and counterparties, financial institutions are required to allocate "economic capital". While value-at-risk (VaR) was traditionally used as the industry standard for capital in risk allocation, *The Basel IV Capital Accord* recommends transitioning from VaR to the expected shortfall (ES) as a more appropriate risk measure during stress situations. Both academics and practitioners support this shift, since the expected shortfall has been identified as the minimally coherent and law-invariant risk measure that supersedes VaR. It is worth noting that the implementation of the expected shortfall (ES) under Basel IV represents a significant evolution in the regulatory framework for market risk management. Basel IV builds upon the enhancements introduced in Basel III, specifically

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through the Fundamental Review of the Trading Book (FRTB), which initially integrated the ES to address the limitations of value-at-risk (VaR) in capturing tail risk (BCBS, 2019). The ES is an informative risk measure reflecting potential losses beyond the VaR threshold, thus offering a better understanding of tail risk (Acerbi & Tasche, 2002a). However, the use of the ES under Basel IV is primarily confined to the applications involving internal models for market risk assessment, as standardized approaches still rely on alternative metrics (BCBS, 2016). This constraint ensures that the implementation of the ES is aligned with the sophisticated risk modeling capabilities of financial institutions while addressing the regulatory need for robust risk quantification and capital adequacy.

Rationales for the supremacy of the expected shortfall are numerous. Value-at-risk represents the maximum potential loss that may happen for a specified threshold probability denoted as p. Losses beyond VaR appear in extreme situations. That having been said, it is clear that VaR denotes the minimum loss in the case of extreme situations that can be anticipated logically. In stress analysis, however, more pertinent information would be the expected loss in extreme situations (which is quantified by the expected shortfall). In addition, the literature acknowledges that VaR lacks coherence because it has no essential attributes of subadditivity and convexity. Consequently, the sum of VaRs from individual portfolios is not necessarily an upper bound of combined portfolios' VaR. Such behavior contradicts the fundamental financial principle of diversification (Artzner, Delbaen, Eber & Heath, 1999; Acerbi & Tasche, 2002a).

In their paper, C. Acerbi and D. Tasche (2002b) introduce a coherent substitute for value-at-risk. The metrics that they proposed are the *expected shortfall* (ES), simultaneously also referring to it as *conditional value-at-risk* in the financial literature. Since it deals with certain drawbacks of VaR, the ES is a more comprehensive risk measure. To be more precise, the financial literature recognizes that the ES offers a more accurate assessment of tail risk, whereas switching from VaR to the ES enhances risk management capabilities. Nevertheless, both of these two tail risk metrics are closely related to one another. The ES is defined as the conditional expected loss given the fact that it surpasses VaR (the case of continuous distributions). More generally, the ES can be delineated as the weighted mean of VaR and the losses beyond VaR (which is suitable for discrete distributions). Empirical studies indicate that minimizing the ES also yields nearly optimal solutions in VaR (as value-atrisk, by definition, never surpasses the expected shortfall). Therefore, portfolios with the low ES have to exhibit low VaR as well. Furthermore, VaR and the ES will be the equivalent metrics in the scenarios where the return–loss distribution is normal (which is rarely the case in real life). In such a case, the portfolio optimizations based on VaR and the ES will both result in the same optimal portfolio. Conversely, the ES and VaR may lead to disparate optimal portfolios for highly skewed distributions. This paper is an attempt to shed more light on the differences between VaR optimal and ES optimal portfolios. In general, VaR cannot be optimized using standard analytical methods. Some research studies show that VaR can be successfully optimized using GA techniques (e.g. Ranković, Drenovak, Stojanović, Kalinić & Arsovski, 2014).

In empirical research studies, genetic algorithms (GAs) have emerged as the preferred method for solving the financial optimization problems that are too intricate for deterministic techniques. The names of these algorithms originate from their execution, which is inspired by the biological (i.e. genetic) processes in the evolution of organisms. Namely, through the evolutionary crossover, mutation and best-fitted individual selection processes, species in nature evolve and become more and more accommodated to the environment. In the same manner, in genetic algorithms, the solutions with the better values of the given objective functions are selected for the recombination (crossover) and mutation, which would more likely result in better offspring solutions in terms of the objective functions. The multi-objective variants of genetic algorithms (MOGAs) are very suitable for solving multi-objective problems since their ability to find a set of optimal solutions (the Pareto front) in a single run, providing a possibility of applying arbitrary constraints on the values of decision variables and/or objective functions (Metaxiotis & Liagkouras, 2012).

In this research study, the multi-objective Strength Pareto Evolutionary Algorithm 2 (SPEA2) algorithm is utilized so as to generate the expected (mean) return–ES and the expected (mean) return–VaR efficient frontiers. To the best of the authors' knowledge, this study explores a relatively under-researched area by investigating the differences in the mean–ES and mean–VaR optimal portfolios obtained by using multi-objective evolutionary algorithms.

LITERATURE REVIEW

The optimization methods based on the processes which mimic natural selection have their roots back in the 1930s. Concurrently, the inclusion of numerous practical constraints in the financial portfolio optimization models have increased their complexity and made them difficult to solve by means of conventional optimization techniques. Those pioneering academic research studies gave rise to the three distinct streams. The insights into each of them will be synthetized in this paper.

The first of three streams includes the studies exploring the utilization of GAs in the realm of portfolio optimization. The research study conducted by S. Arnone, A. Loraschi and A. Tettamanzi (1993) investigated the bi-objective optimization problem in the context of the mean return-variance-based risk measures for unconstrained optimization. Having introduced the trade-off coefficient, the authors converted the initial bi-objective problem into a single-objective one. Similar research was carried out by T.-J. Chang, S.-C. Yang and K.-J. Chang (2009) and E. P. Setiawan and D. Rosadi (2020), who proposed the GA-based mean return-risk optimization under the cardinality constraint. The authors considered risk measures such as semi-variance, the mean absolute deviation and variance with skewness. When optimization itself is concerned, the authors followed the procedure for the transformation of the bi-objective problem to the single-objective problem via the trade-off coefficient having been proposed by S. Arnone *et al* (1993). In contrast, V. Ranković *et al* (2014) explored the GA-based portfolio optimization with historical VaR as a risk metric. In their paper, the authors introduced two distinct optimization approaches, namely the single-objective approach, by employing the single-objective GA (SOGA), and the multi-objective approach, by employing the multi-objective GA (MOGA). Both methods provide the mean-VaR efficient frontiers that exhibit favorable risk/reward trade-offs for solution portfolios. However, the authors stress that the MOGA approach outperforms the SOGA counterpart in terms of computational efficiency.

The second stream found in the literature addresses the challenge of portfolio optimization that utilizes the ES as the measure of portfolio risk. The relevance of the ES as a risk measure can be best illustrated by the fact that it transcended traditional financial analysis which it originates from. It has gained traction in diverse fields of science and the academic literature, such as breast cancer therapy (Chan, Mahmoudzadeh & Purdie, 2014), scheduling (Sarin, Sherali & Liao, 2014), and machine learning (Takeda, 2009; Takeda & Kanamori, 2009; Takeda, Fujiwara & Kanamori, 2014; Wang, Dang & Wang, 2015). There are attempts to tackle the problem using the GA to optimize the portfolio regarding the different versions of the ES (Wang *et al*, 2015; Jadhav & Ramanathan, 2019).

C. Acerbi and D. Tasche (2002a) outlined a methodology for the assessment of the ES risk contribution of individual portfolio constituents. S. Ciliberti, I. Kondor and M. Mézard (2007) explored the feasibility of portfolio optimization under the ES as a risk measure. Their paper demonstrates the fact that, if the asset-to-data point ratio (i.e. N/T) exceeds the critical value, empirical return distributions are not defined well. Since the critical value is contingent upon the ES probability threshold, the lower it is, the longer time series is required for effective portfolio optimization. F. Caccioli, J. D. Farmer, N. Foti & D. Rockmore (2015) proposed a novel approach to determining the requisite length of time series for the effective portfolio optimization based on the ES as a risk metric, their approach relying on the construction of contour maps. The maps are constructed based on the confidence level, the relative estimation error and the number of portfolio constituents. Their findings suggest that the requisite sample size is often unfeasibly large for rational portfolio optimization scenarios. In other words, effective portfolio optimization would require unreasonably long time series of returns, regardless of the chosen confidence level and the number of constituents in the portfolio.

The third and the last stream in the literature delves into the diverse methodologies employed by researchers when utilizing GAs in solving complex portfolio optimizations. One such paper is that by C.-C. Lin and Y.-T. Liu (2008), who conducted a research study focusing on the seminal H. Markowitz (1952) portfolio optimization model that incorporates a constraint on minimum transaction lots. Using SOGA, researchers derived the mean return-variance efficient frontiers. D. W. Corne, J. D. Knowles and M. J. Oates (2000) carried out another important research study and demonstrated the exceptional performance of SPEA in comparison to the other MOGAs. Therefore, SPEA has been established as the generally accepted benchmark in many recent academic research studies on this topic. Building upon this foundation, E. Zitzler, M. Laumanns and L. Thiele (2002) introduced the enhanced version of SPEA known as SPEA2. The improvements include the refined archive truncation method, the addition of the density-estimation technique, and the new fine-grained refined fitness assignment strategy. Thanks to them, SPEA2 dominates its predecessor. The papers by K. P. Anagnostopoulos and G. Mamanis (2011) and V. Radak (2020) will also be addressed. The authors further explored and compared the GA portfolio optimizations based on the mean-variance, mean-ES, and mean-VaR methodologies, which include quantity, cardinality, and class constraints. Their research revealed that the Non-dominated Sorting Genetic Algorithm II (NSGA-II), the Pareto Envelope-based Selection Algorithm (PESA), and SPEA2 exhibited efficient performance regardless of the used risk metric.

MORE ON THE EXPECTED SHORTFALL

Here, A. J. McNeil, R. Frey and P. Embrechts (2015) definition of the ES is presented. For the loss L whose distribution function is F_L and which has its finite expected value $E(|L|)<\infty$, the ES at the confidence level $\alpha \in (0,1)$ is defined as follows:

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_u\,(F_L)du \tag{1}$$

where $q_u(F_L)=F_\tau(u)$ is the quantile function which depends on the distribution function of the loss L, i.e. F_L. For the given value of VaR_α, ES_α, can be obtained as follows:

$$ES_\alpha = E[X|X \geq VaR_\alpha(X)] \tag{2}$$

The equation (2) makes it clear that the ES essentially represents the expected loss which surpasses VaR, i.e. the anticipated loss in extreme scenarios. As was pointed out by C. Acerbi, C. Nordio and C. Sirtori (2001), the ES can easily substitute VaR as the downside measure, given the fundamental similarities between the two approaches, which is also true for any other left-tail statistics. Nevertheless, the ES still has some drawbacks. One notable limitation pointed out by Y. Yamai and T. Yoshiba (2005) implies its substantial susceptibility to estimation errors. Consequently, it requires very long time series of returns for robust estimation.

To define the ES within the scope of portfolio optimization, the portfolio loss needs to be defined. that the portfolio loss is assumed to be the function $L(x,y)$. It depends on the two vectors: x and y. Vector x denotes the vector of unknown portfolio weights. Vector y is the random vector characterized by the probability density function $p(y)$ which represents uncertainties in the market parameters influencing the loss. Consequently, the probability that $L(x,y)$ falls below a threshold value β is going to be a function $\psi(x,\beta)$. Under this setup, the portfolio's ES for the loss function corresponding to the vector of weights x at the given confidence level $\alpha \in (0,1)$ can be expressed as follows:

$$\phi_\alpha(x) = E[\mathcal{L}(x,y)\,|f(x,y) \geq \beta_\alpha(x)]$$
$$= \frac{1}{1-\alpha}\int_{\mathcal{L}(x,y)\geq\beta_\alpha(x)} \mathcal{L}(x,y)p(y)dy \quad (3)$$

The previous notation is inadequate for practical implementation. Therefore, the following reconstruction is recommended, namely:

$$\phi_\alpha(x) = \min_{\beta\in R} F_\alpha(x,\beta) \quad (4)$$

where

$$F_\alpha(x,\beta) = \beta + \frac{1}{1-\alpha}\int_{y\in R^m} [\mathcal{L}(x,y)-\beta]^+ p(y)\phi \quad (5)$$

and $t^+=max\{t,0\}$.

The utilized portfolio optimization models

The variation of H. Markowitz's (1952) multi-objective ES-based optimization model is introduced at this point. The presented model aims to simultaneously maximize the portfolio's expected return (denoted as $\mu_p(x)$ in (Eq. 6.2)) and minimize the portfolio's risk estimated via the expected shortfall (denoted as ES in (Eq. 6.1)).

$$min\ \ ES_\alpha(x) \quad (6.1)$$

$$max \sum_{j=1}^{n} r_j x_j = \mu_p(x) \quad (6.2)$$

$$s.t. \sum_{i=1}^{n} x_j = 1, \quad (6.3)$$

$$0 \leq x_j \leq u_j, \qquad j = 1,\dots,n. \quad (6.4)$$

The equation 6.3 is a budget constraint. It ensures that the sum of weights must be equal to 1, i.e. that the entire investment budget must be invested, nothing more and nothing less. The equation 6.4 represents the short-selling prohibition and the holding constraint (i.e. it puts a limit on the weight of the budget that can be invested in a single asset). Mathematically, it prohibits negative weights and limits the maximum size of each weight. Unless otherwise stated, u_j is equal to 1.

THE STRENGTH PARETO EVOLUTIONARY ALGORITHM 2 (SPEA2)

The Strength Pareto Evolutionary Algorithm 2 (i.e. SPEA2) introduced in the paper by E. Zitzler, M. Laumanns and L. Thiele (2002) is the multi-objective evolutionary algorithm that seeks the exact or approximate Pareto-optimal set of solutions. It is the improved version of the original version of SPEA developed by E. Zitzler and L. Thiele (1999). In contrast to the single-objective counterpart, the multi-objective algorithms such as SPEA2 create the Pareto-optimal solution set in a single run.

The main loop is given as follows:

Step 1. The initialization: Generate the initial population P_0 and create the empty archive $\overline{P_0} = \emptyset$. The set t=0.

Step 2. Fitness assignment: Calculate the fitness values of the individuals P_t and $\overline{P_t}$. The fitness value $F(i)$ of the individual i is defined as follows:

$$F(i) = R(i) + D(i) \quad (7)$$

where $R(i)$ denotes raw fitness and $D(i)$ denotes density. Raw fitness is calculated as follows:

$$R(i) = \sum_{j\in P_t+\overline{P}_t,\, j\succ i} S(j) \quad (8)$$

where $S(j)$ denotes the strength value of the individual j, and represents the number of solutions in the actual population and the archive that are dominated by the solution j:

$$S(j) = |\{m|m \in P_t + \overline{P}_t \wedge j \succ m\}| \quad (9)$$

$|\cdot|$ describes the cardinality of the set, + means the multi-set union and $\succ$ resembles the Pareto-dominance.

The density value $D(i)$ is defined as the function of the distance to the k-th nearest solution (σ_i^k):

$$D(i) = \frac{1}{\sigma_i^k + 2} \quad (10)$$

where $k = \sqrt{N + \overline{N}}$, N is the population size and $\overline{N}$ is the archive size.

It is important to note that the fitness value is to be minimized.

Step 3. Environmental selection: Copy all the non-dominated individuals in P_t and $\overline{P_t}$ to $\overline{P_{t+1}}$. If the size of $\overline{P_{t+1}}$ exceeds $\overline{N}$, reduce $\overline{P_{t+1}}$ by means of the truncation operator; otherwise, if the size of $\overline{P_{t+1}}$ is less than $\overline{N}$, fill $\overline{P_{t+1}}$ with the dominated individuals in P_t and $\overline{P_t}$.

Step 4. Termination: If $t > T$ or if another stopping criterion is met, set A to the set of the decision vectors represented by the non-dominated individuals in $\overline{P_{t+1}}$. Stop.

Step 5. Mating selection: Perform binary tournament selection with replacement on $\overline{P_{t+1}}$ in order to select the parents. Binary tournament selection implies a random selection of two solutions from a given set, and the solution with a better fitness value is selected for mating. The process is repeated until the mating pool is completed.

Step 6. Variation: Apply the recombination and mutation operators to the mating pool and set $\overline{P_{t+1}}$ to the resulting population. Increment generation counter (t->t+1) and move on to Step 2.

In the case of the SPEA2 algorithm, the fitness value is complex and combines the three values: the number of the solutions dominated by the given solution, the number of the solutions dominating the given solution, and the density value that measures the distance from the other solutions in the solution set. Lower density values are preferred.

DATA SOURCES AND THE MAIN COMPUTATIONAL RESULTS

In this section, the empirical results obtained by conducting optimization experiments on the daily historical returns of the DAX constituents were subjected to analysis. The sample spans from 5th January 2015 to 28th April 2017. A total of seven distinct assets were selected for further investigation due to their favorable distribution in terms of both risk and return (see Figure 1). The reason for a relatively small number of assets lies in the computational times. The selection of the seven German stocks over a two-year period appears to be unconventional to demonstrate the method. This sample size is limited both temporally and cross-sectionally. The reason for that lies in the computational times: the latest run for the tests that include the SPEA2 algorithm took more than four days. An extended set of stocks would exponentially have extended the computational times and would have presented a significant challenge in making investment decisions in the long run. A broader dataset, encompassing a more extensive range of stocks over the entire available historical period or including various asset classes and markets, would have provided a more robust illustration. The selection of the data used for this illustration should be justified and whether their methodology has the computational constraints necessitating such a restricted sample size should be clarified.

Based on the previous results, as many as seven assets were selected for portfolio optimization, namely *Münchner Rückversicherungs-Gesellschaft AG*, *Beiersdorf AG*, *Henkel AG & Co. KGaA*, *Siemens AG*, *Deutsche Börse AG*, *Fresenius SE & Co. KGsA*, and *Infineon Technologies AG*, the review of whose expected returns and risks are accounted for in Table 1. Risk is measured as a 5% 1-day historical ES. When the expected returns are concerned, they were computed as the mean of the daily asset log returns. However, the expected returns are not annualization. Their annualization would create a significant difference in the level between the risk and expected return for the selected asset, which would make optimization more computationally difficult. Instead of that, the daily log returns were scaled to a monthly basis, so that the levels of the expected returns and risks were nearly in the same range.

Table 1 The review of the risk (ES) and expected log return for the selected assets

Number	Asset	ES	Mean log returns
1	Münchener Rück.	2.71%	0.98%
2	Beiersdorf AG	2.87%	1.50%
3	Henkel AG	2.98%	1.73%
4	Siemens AG & Co. KGaA	3.31%	2.16%
5	Deutsche Boerse AG	3.46%	2.24%
6	Fresenius SE	3.55%	2.97%
7	Infineon Technologies AG	4.22%	4.28%

Source: Authors

Multi-objective optimization with SPEA2

The results obtained using the SPEA2 genetic algorithm described in Section 4 of the paper are presented here. The algorithm was executed twice with different settings. In both trials, the population size was set to 500 individuals, while the number of iterations was set to 200. As is illustrated in Figures 2 and 3 (which depict the optimization results), the archive size was designated to 21 in the first trial, whereas in the second, it was adjusted to 250.

As is shown in Figure 2, SPEA2 produces optimal portfolios with a superior return-to-risk ratio compared to the individually analyzed assets. To ensure the presence of the maximum return solution, which is always a single-asset solution, this single-asset solution was added in the initial population. The exact results obtained via the first optimization trial are given in Table 2.

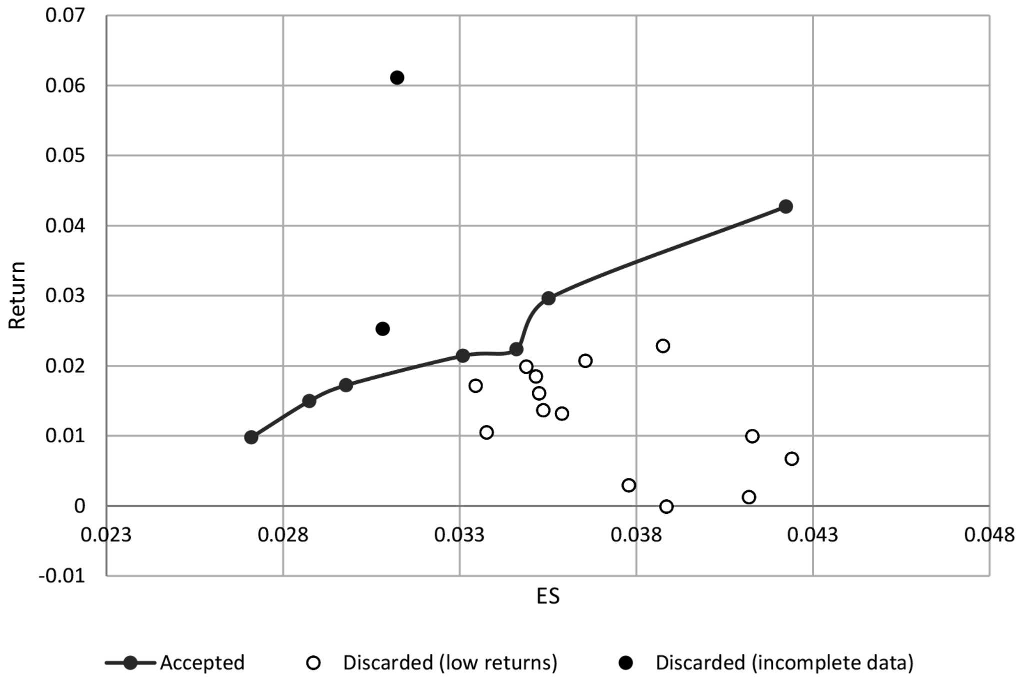

Figure 1 The summary account of risk/return for all the DAX constituents, plotted as the mean daily log return – the ES efficient frontier

Note: The white dots represent the constituents that were discarded due to the relatively low returns compared to the ES, whereas the black dots represent the constituents that were discarded due to the incompleteness of the data.

Source: Authors

Table 2 The optimization results obtained by using SPEA2 with the archive size 21 in the first trial

Number	Mean	ES	Number	Mean	ES
1	2.21%	-2.63%	12	3.31%	-3.14%
2	2.25%	-2.64%	13	3.34%	-3.16%
3	2.40%	-2.70%	14	3.45%	-3.25%
4	2.55%	-2.76%	15	3.56%	-3.32%
5	2.61%	-2.77%	16	3.65%	-3.41%
6	2.78%	-2.84%	17	3.73%	-3.45%
7	2.97%	-2.94%	18	3.82%	-3.53%
8	3.02%	-2.97%	19	3.93%	-3.65%
9	3.07%	-2.99%	20	4.03%	-3.83%
10	3.13%	-3.04%	21	4.26%	-4.20%
11	3.19%	-3.06%			

Source: Authors

The results obtained from the second optimization trial are plotted in Figure 3, which clearly shows that the largest number of the optimal portfolios obtained in the second trial are concentrated around the middle and the maximum expected returns. It should be noted that, in the second optimization trial, optimal portfolios with a lower risk level compared to those obtained in the first optimization trial can be seen. As before, however, it seems that there is a lack of optimal portfolios around the minimum risk (see Figure 3).

Comparison with the mean-VaR optimal portfolios

To analyze the differences between the results obtained in the case when VaR is set as the optimization objective instead of the ES, the experiment was repeated once more, but now with VaR as a risk metric. The results obtained with SPEA2 are depicted in Figure 4.

Figure 4 illustrates the distribution of the optimal portfolios obtained by SPEA2 with the VaR minimizing objective across the efficient frontier, showing simultaneously a superior return-to-risk ratio compared to the seven selected individual assets. If Figure 3 is contrasted with Figure 4, however, it can be seen that there is one notable difference between the optimal portfolios (i.e. the efficient frontier) produced by SPEA2 which minimizes the ES (as is depicted in Figure 3) and those obtained when the same algorithm minimizes VaR (as is depicted in Figure 4). The resultant "efficient" frontier in the case of VaR minimization appears to be flatter and, even after the optimization iterations had doubled, it was still impossible to make any significant improvement. Furthermore, the research results suggest that the optimal portfolios for the lower levels of the targeted expected returns were not distributed well and seemed to be limited by a barrier. These disparities were best seen when the optimal portfolios obtained

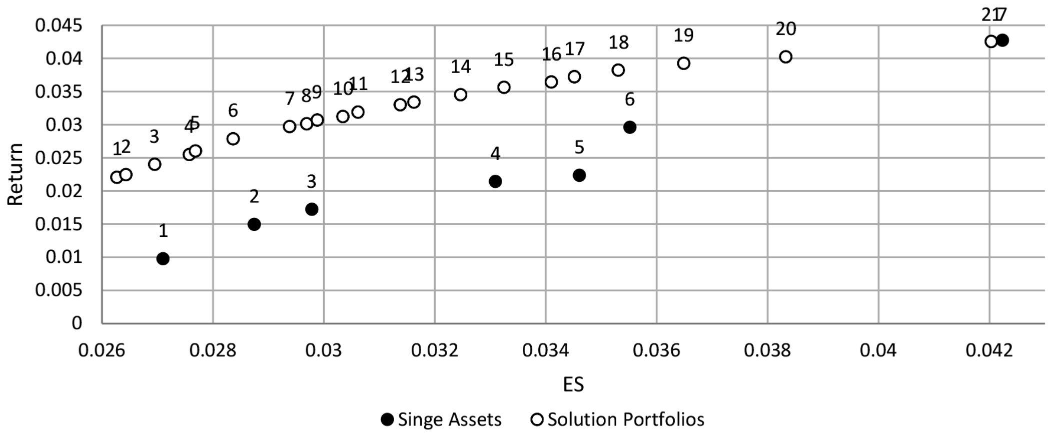

Figure 2 The results of SPEA2 for the archive size 21

Source: Authors

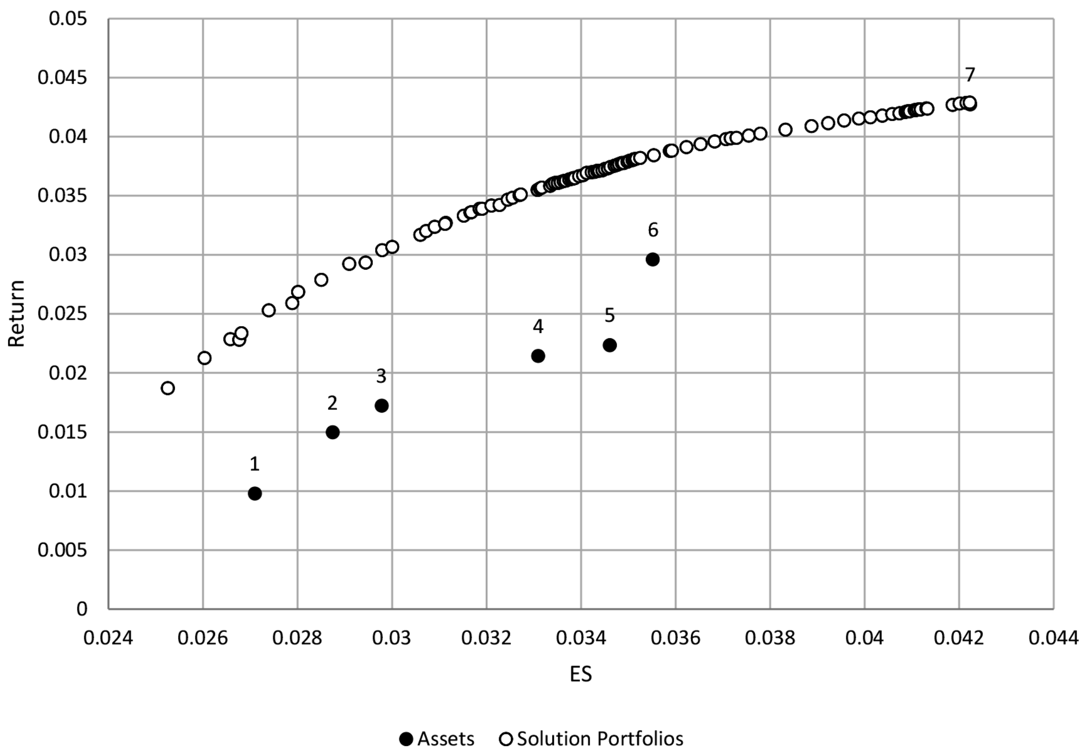

Figure 3 The optimization results obtained using SPEA2 with the archive size 250 in the second trial

Source: Authors

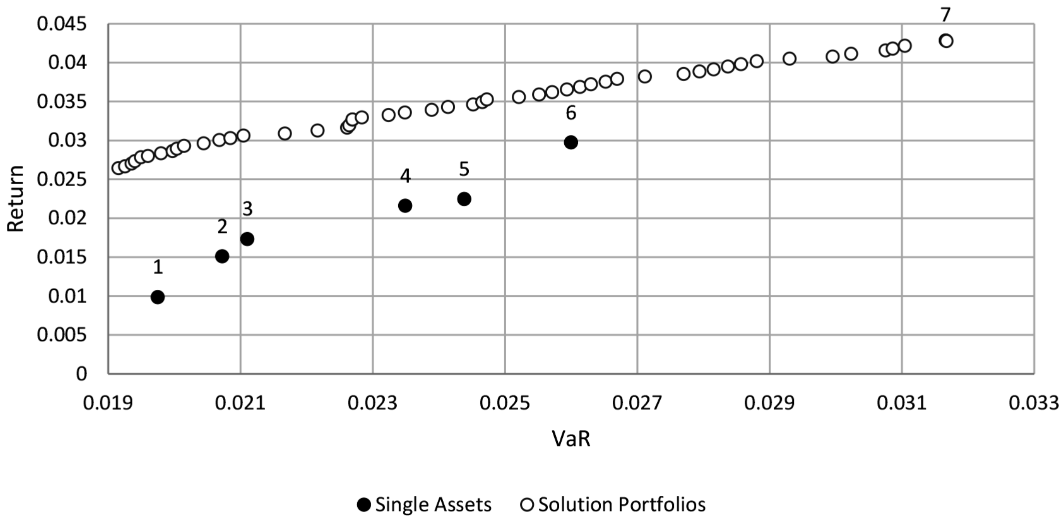

Figure 4 The SPEA2 optimal portfolios based on VaR as a risk objective vs the individual assets

Source: Authors

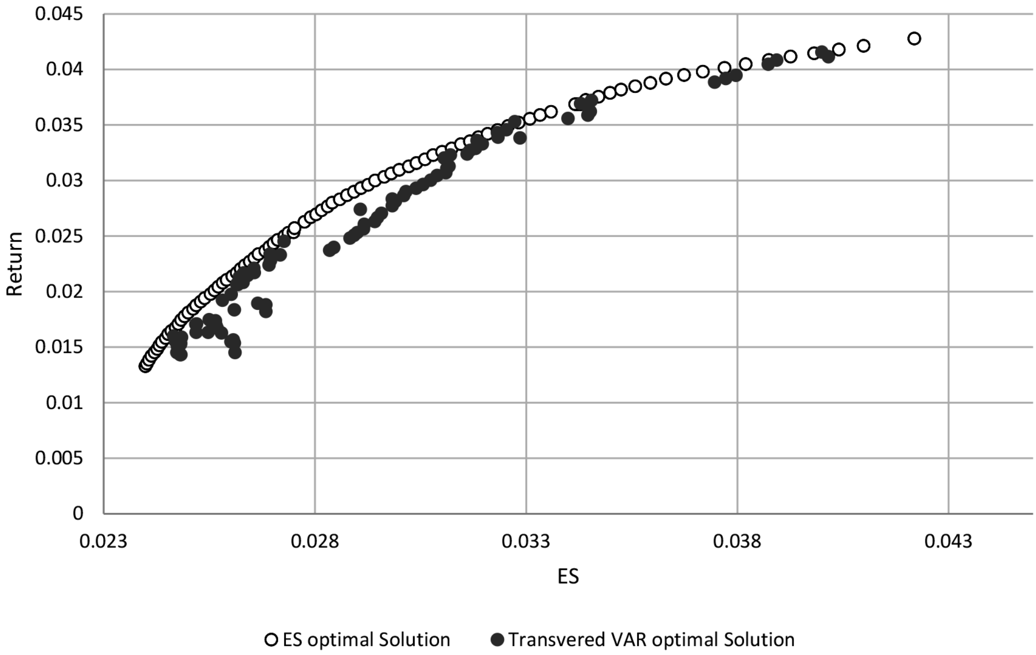

Figure 5 The transformed VaR optimal portfolios vs the ES optimal portfolios

Source: Authors

by mean-VaR optimization were plotted against those obtained by mean-ES optimization in the mean–ES plain. To do so, the ES was computed for the mean-VaR optimal portfolios, and the plotted result is displayed in Figure 5.

According to Figure 5, the mean-VaR optimal portfolios are evidently not distributed well when transformed in the mean-ES plain. The absence of solutions in the ES range of approximately 3.5% to 3.7% is apparent, with the more pronounced clustering of the optimal portfolios with the low expected return (considering the left-hand side of the graph depicted in Figure 5). In addition, the numerous portfolios exhibit nearly identical expected returns, yet display notable discrepancies in the ES values. These results were the motivation to compare the previous two sets of the optimal solutions in the mean-VaR plain as well. To do so, the VaR of the mean-ES optimal portfolios were computed and then those portfolios were plotted against the mean-VaR optimal portfolios in the mean-VaR plain (Figure 6).

As can be seen in Figure 6, the mean-ES optimal portfolios are distributed significantly better along the resulting efficient frontier. In addition, it can be noted that both efficient frontiers now look very similar to each other. The optimal portfolios obtained via ES minimization seem to converge towards the VaR optimal portfolios and nearly coincide for the high levels of the expected return.

In the end, a conclusion can be drawn that, generally speaking, VaR optimization does not provide ES optimal, or near-optimal, solutions. ES optimal solutions may simultaneously generate near-optimal VaR solutions. Consequently, in the case when both the low ES and low VaR are the desirable properties of the managed portfolio, it may be worthwhile to optimize it with respect to the ES.

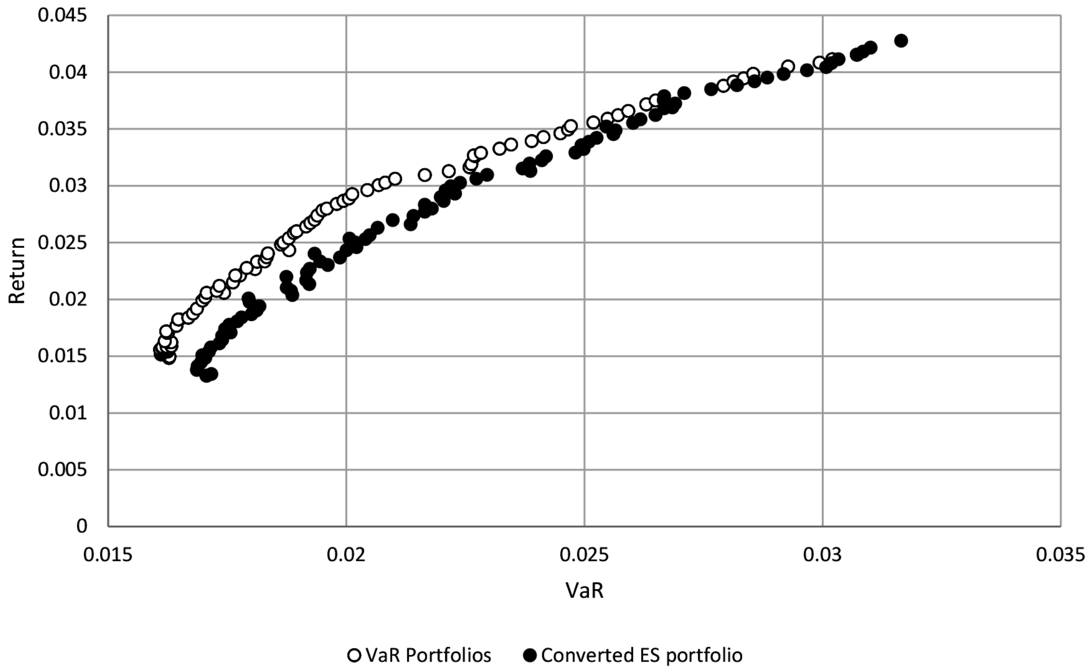

Figure 6 The transformed ES optimal portfolios vs the VaR optimal solutions

Source: Authors

CONCLUSION

This research study explores the applicability of the expected shortfall as a risk measure in the optimal portfolio selection problem. To generate optimal mean-ES portfolios, SPEA2 introduced by E. Zitzler *et al* (2002) was employed. Aiming to obtain computational efficiency, as many as seven most efficient assets from the DAX index were opted for.

In order to establish a benchmark for the results of the research study, SPEA2 was also used to generate the mean-VaR optimal portfolios. The efficient frontier obtained by mean-VaR optimization showed a flatter profile compared to that generated by the ES and produced the well-dispersed optimal solution. Subsequently, the ES for the mean-VaR optimal portfolios was computed and those portfolios were plotted with the mean-ES optimal portfolios. Notable disparities were observed between the two frontiers, particularly at the lower expected return levels. The transformation of the mean-VaR solutions to the mean-ES plain revealed the uneven distribution and absence of certain ES values. Conversely, VaR for the mean-ES optimal portfolios was also estimated and those portfolios were plotted against the mean-VaR optimal portfolios. The resulting efficient frontiers closely resembled each other, with the mean-ES solutions aligning well with the mean-VaR optimal solutions. As a matter of fact, it seems that the mean-ES optimal portfolios converge to the mean-VaR optimal solutions. The findings of this research study demonstrate that the portfolio optimization based on the minimization of VaR and the ES can produce significantly different optimal portfolios for the same opportunity set of assets.

The shortcomings of the no-short-selling constraint are nevertheless acknowledged. Short positions can be strategically employed through direct hedges in asset-liability management in order to manage various exposure types. Therefore, omitting this constraint limits the applicability of this study and limits its ability to capture the full spectrum of real-world portfolio management practices, potentially reducing the relevance and robustness of the findings.

The limitations of this study are also recognized and avenues for future research in this domain are identified. Firstly, the number of the assets chosen for the optimization algorithms is very small and random. Due to the simple structure of SPEA2, it would be easy to add certain real-world constraints, such as cardinality constraints or short selling. Furthermore, a more in-depth comparison with different multi-objective evolutionary algorithms would be of interest as well.

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